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How concepts of mathematics in primary education function as ausubel's advance organizers and scaffolding to overcome bachelard's cognitive obstacles

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Abstract
Certain concepts in mathematics may function as cognitive obstacles for the students of secondary, or even those of tertiary education. These concepts are difficult for them to understand. In this paper, we highlight the concepts of epistemological, cognitive, and didactical obstacles, and we explain the significance of each one them, and how they arise, and how they can be bypassed. Moreover, we suggest that the adoption and use of these fundamental concepts of mathematics in the teaching-learning process early in the primary education, can definitely function as an advance organizer or as a means of scaffolding. According to the theory of Ausubel, this process will facilitate students' attempts to overcome the problem of not understanding these concepts. Finally, we refer to examples of such concepts, and useful conclusions are made.

Keywords: mathematics education, primary education, cognitive obstacles, Ausubel theory, advance organizers, scaffolding

1. Introduction

Experienced teachers and educational researchers both corroborate that students find it quite difficult to understand some certain concepts of mathematics, especially the new and uknown ones. Meanwhile, it has been observed that students make some systematic mistakes during the teaching of these mathematical concepts. For example, students when being in the third class of greek junior high-school (herein, we call it Gymnasium), they often make the following mistake: $(a+b)^2=a^2+b^2$. What really urges students to make systematically similar mistakes like the one mentioned before? What really hinders students from understanding certain concepts in Mathematics? According to Sierpinska (1999) [12], students' minds are not "tabula rasa", at the moment they start learning a new mathematical concept. Instead, their minds are full of prior knowledge, beliefs and experience, hence the newly acquired knowledge is not just added to the previous knowledge, but in most cases it must be merged with them. Moreover, it is not unusual for the new knowledge to come to a contradiction with the previous knowledge, and then the previous knowledge may act as an obstacle to the grasping of the new one. The term "misconception" in mathematics occurred in the USA in 1981, in a text by Wagner (1981) which treated the learning of equations and functions. Again in 1981, a celebrated text by Kieran (1981) [15]. discussed equation solution activity. 1982 saw the publication of an article belonging to the algebra learning domain: Clement (1982). In 1983 we have works by Wagner (1983) and Kieran (1983), again on algebra. Then numerous works published in 1985 specify the term "misconception": Schoenfeld (1985) [9], Shaughnessy (1985) [9]. and Silver (1985) [11, 13], who use it mainly with regard to problem solving, together with beliefs or to explain their interactions. In Silver (1985, pp. 255-256) [11, 13]. it is stated explicitly that there is a strong tie between misconceptions and mistaken beliefs. Schoenfeld (1985, p. 368) highlights how students can correctly develop some incorrect conceptions, particularly regarding procedures. The concept of misconception was not precisely defined at the moment of its entrance into the world of Mathematics Education research, but, as we have seen, it was, and still is, used with its intuitive meaning (D'Amore & Sbaragli, 2005) [6].

2. The concept of 'obstacles'

The term "obstacle" was coined by Bachelard (1938) [2], who sees the obstacle as an internal structure that prevents objective thinking and also he points out that it represents a way of dealing with new knowledge in relation to the pre-existing one, i.e that the new knowledge is treated as a controversial concept to that of pre-existing knowledge. Bachelard (1938) [2] notes:

"We have to put the problem of scientific knowledge in terms of obstacles. It is not enough to solely consider external obstacles, such as the complexity and temporality of scientific phenomena, nor to mourn the weakness of the human senses and spirit. It is part of the action of acquiring one's own knowledge, to know deeply what is appearing, as an inevitable result of the functional need, to delay the speed of learning and to cause cognitive difficulties. Here, we are able to find the causes of this stagnation and even of this regression and thus we may realize the true reasons for students' inaction, which we call "epistemological obstacles ".

And he continues: "We are dealing with new knowledge that contradicts pre-existing knowledge and so we have to destroy pre-existing misconceptions."

He also argues that epistemological obstacles appear during the historical development of scientific thought and in educational practice and are therefore distinguished by two essential features:

- They are inevitable and essential components of the

acquired knowledge
- They contribute, at least partially, to the historical development of the concept.

From a cognitive perspective, i.e the view that the student is capable of reproducing his own knowledge, the errors are a kind of knowledge, deeply rooted in student's consciousness that prevents him them from the understanding of the new knowledge. Brousseau (1997) [3] notes that an obstacle does not represent a lack of knowledge, instead it is considered as a knowledge, which produces appropriate answers within a frequently answered but limited framework and which is not generalizable beyond this operating framework. In other words, it fails to function satisfactorily in another context, thus leading to contradictions. Moreover, D'Amore and Sbaragli (2005) [6]. examined the interpretation of the reality of the subject, and concluded that students' interpretation is merely created on the basis of personal convictions which have matured partly as a result of some kind of learning; therefore, it is absolutely rational to view misconceptions as the fruit of something known, not as an absolute lack of knowledge. In this paper we will briefly refer to the epistemological, cognitive and didactic obstacles.

2.1 Epistemological obstacles

Bachelard (1938) [2]. viewed epistemological obstacles as a very useful tool for studying the history of science, and in particular its pre-scientific era. Indeed, epistemological obstacles are related to the history and evolution of science and according to Bachelard (1938) [2], they are also related to the process of knowledge development. These obstacles are inherent in knowledge itself. Epistemological obstacles to the history of mathematics, as a science, can be traced back to the difficulties historically encountered by mathematicians themselves and their attempts to overcome these difficulties.

Bachelard (1938) [2]. proposed that the history of science is replete with "epistemological obstacles" - or unthought/unconscious structures that were immanent within the realm of the sciences, such as principles of division (e.g., mind/body). The history of science, Bachelard asserted, consisted in the formation and establishment of these epistemological obstacles, and then the subsequent tearing down of the obstacles. This latter stage is an epistemological rupture—where an unconscious obstacle to scientific thought is thoroughly ruptured or broken away from. Hence, it represents the basis of epistemological obstacles. This explains why Bachelard suggests that we think of science more in terms of disruption, than in terms of continuity.

Duroux, as Koleza (2000) states, believes that an epistemological obstacle is characterized by the following four characteristics:

1. It is a knowledge with a fairly wide scope of application.
2. This knowledge, while trying to be applied to various situations, it causes errors identified and analyzed only in relation to the obstacle.
3. The obstacle resists the attempt of its specialized application.
4. Rejection of knowledge (which was considered as an epistemological barrier) creates new knowledge or, as Bachelard writes, we learn against an older knowledge.

Epistemological obstacles are related to the history and evolution of science. According to Bachelard (1938) [2]. epistemological obstacles are a kind of obstacles related to the process of knowledge development, historically found by the mathematicians themselves and their attempts to overcome them. The understanding of these obstacles is enriched by the research in the fields of epistemology and history of mathematics. Furthermore, when they were first observed, they were classified as "paradoxical" and they were not included in the strictly defined area of pure mathematical research. Ultimately, as such phenomena, they proved to be the driving force and the principal cause of great evolution in mathematics. The following are some typical examples:

1. The concept of 'infinity' is an example of an epistemological obstacle, because its historical evolution that represents a source of great difficulty for the foundation, is full of endeavors for its definition. These attempts range from the paradoxes of Zeno of Eleatis till those of Cantor and Russel.
2. The concept of 'zero' is also a typical example of an epistemological obstacle. Although it was first appeared in India to fill in the gaps of a positional numeral system, though one had to wait until the 9th century, when it can be found again in Arabic essays. Moreover, in the 12th century it becomes clear how its algebraic properties are processed.
3. The concept of 'function' is another one example of an epistemological obstacle required two thousand years to establish itself in today's reality.
4. The concept of 'limit' that according to Cornu (1991) four important epistemological obstacles appear, throughout its historical evolution:

- The failure to connect geometry to numbers.
- The concept of infinitely large and that of infinitely small.
- The metaphysical view of the concept of limit.
- Can we reach the limit or not?

2.2 Cognitive obstacles

Cognitive obstacles are the equivalent of epistemological obstacles at the individual level and therefore we use the term 'cognitive obstacles', when referring to students. The concept of cognitive barrier is interesting to study, to help us identify the difficulties encountered by students during the learning process and to identify appropriate strategies for teaching. For example, using the historical development of the function, it has been ascertained that an epistemological obstacle that students must overcome is the concept of the function as an expression, just as happened with Euler (Sierpinska, 1992).

2.3 Didactical obstacles

Didactical obstacles derive from the formalist instructional method, as well as from the traditional teacher-centered instruction, where the students are usually not given the opportunity to relate the recently taught concepts with their prior knowledge. This discontinuity may represent a critical source of numerous obstacles. This kind of obstacles may also arise during the phase of didactical metaphor, i.e when presenting a complex subject that is usually rationalized on the basis of principles such as 'from simple to complex' or 'from partial to general', etc. The result is to appear additional didactic obstacles.

3. Viewing Ausubel's Advanced Organizers as a Means of Scaffolding

The role of "scaffolding" is extremely substantial, regardless of the learning theory it might be used for. Actually, when the computer is involved in the teaching process by the instructor, then the benefits of the scaffolding may be viewed even in the case of traditional teaching. Indeed, Nikoloudakis & Choustoulakis (2007) introduced a mathematical teaching software that enabled students to choose their preferred learning method, among three available, ready-to-use learning theories contexts: that of constructivism, of socio-cognitive theory, and that of traditional teaching. Johnson, Johnson and Stanne (1995) after having conducted significant research on collaborative learning using Information and Communication Technologies (ICT), they reached to the following conclusions:

1. Computer-assisted collaborative learning helps to achieve a) more quantity and higher quality of daily attainment, b) higher capacity of documented learning, and c) optimized capacity for the students to use their knowledge in problem solving.
2. Collaborating teams are faster and more accurate than the individualistic and competitive ones.
3. Students tend to need less help from the teacher.

The appropriate classroom setup when it comes to implement educational software activities is to form groups of two or three students. In this way students emphasize on the group study, on the discussion of complex ideas or on the elaboration of difficult steps and proposed topics. Besides, most students feel very excited when involved in learning activities that allow them to interact and communicate with their classmates. Ausubel (1963) describes advance organizers as a "mental scaffolding" of the new knowledge, and he is not concerned, if the acquired knowledge is conquered through discovery or if it is given almost ready to use.

However, here we corroborate that we can use advance organizers as a means of 'mental scaffolding', by ensuring the necessary conditions for the student to discover knowledge, so that the member of a team to be able to discover the knowledge by his own. Hence, the advance organizers are necessary for us not for the purpose of offering prefabricated knowledge, but for offering knowledge that can be discovered by the student. For example, if the teachers use the CmapTools software to create advance organizers that will facilitate them in the conceptual development process, this will not be implemented by themselves, instead it will be implemented by each one of the student groups, that will appropriately enrich the concept maps, e.g. with images and other related material. It should be noted that the use of concept maps and diagrams in general promotes participation and collaborative learning, while it simultaneously increases students' learning comprehension.

4. Examples

4.1 The integer part of a real number

A concept often misunderstood by the students is that of the integer part of a real number. By definition, the integer part of a real number is the largest integer that does not exceed the number. On the one hand, the words 'larger' and 'exceed' and, on the other hand, the symbolic mathematical language used, represent certain obstacles that hinder the understanding of the concept of the integer part of the real number by the students. The phrase 'does not exceed' is conceived by the students as something that it cannot be approached because it is larger, while the use of the adjective 'larger' also appears in the definition of the real number. This merely causes a confusion in students' mind. This confusion is also complemented by: a) the inequality $n \leq x \leq n+1,\ n \in \mathbb{Z}$ and, b) the equality {x}=x-[x], that make the situation even worse. Particularly, when they are given a positive real number e.g. 3.4 and they are asked to write down the integer part of this number, they write [3.4]=3, but when they are given a negative real number e.g -3.4, they write [-3.4]=-3, that is definitely false.

In this paper we refer to the teaching of decimal integers in primary education. Indeed, students in primary education come across positive only numbers, so they are taught to separate the integer part from the decimal part of a real number using a decimal separator, which is the dot "." in many countries (including all English speaking ones). In Greece the decimal separator used is a comma "," and also in other countries (mainly in continental Europe). It is exactly this situation that leads students to write the false relationship [-3.4] = -3. However, if the teacher in the Gymnasium or Lyceum refers to this knowledge, emphasizing that the integer part of the number in the primary school was the immediately smaller integer number before the given decimal number, then the students will understand that [-3.4]=-4. The most critical characteristic of teacher's instruction, that will also play the role of an advance organizer, according to Ausubel, is not only the emphasis on how to separate the integer part of the number from its decimal, but also the emphasis on the fact that the integer part of the number is the immediately smaller integer number before the given number. This can be effectively taught in the following way: the teacher asks the students to draw an integer number line, containing only the positive integers, including number zero (0). Then, students are asked to mark the number "4.32". After this, students are asked to observe carefully, between which numbers, the number "4.32" is located. Then, the teacher asks the students which one of these numbers are the smallest one. Students answer that the smaller number is number "4". At this point, the teacher writes on the classroom blackboard, the integer part of the number "4.32", i.e number "4", using red color pen, and also writes the decimal part of the number "4.32" i.e number "32", using white color pen. Teacher and students jointly decide that number "4", being the immediately smaller number before the number "4.32", is the integer part of the number "4.32". This can also be effectively taught, if the teacher asks from the students to write down all the integer numbers that are immediately smaller than number "4.32", and then to select the largest among them. Specifically, teachers in primary education It should be clarified, already from the primary school, that there also exists an integer part of the integer number, and this equals the number itself, e.g. [5]=5, so this knowledge will constitute an advance organizer for understanding also the relationship [-5]=-5.

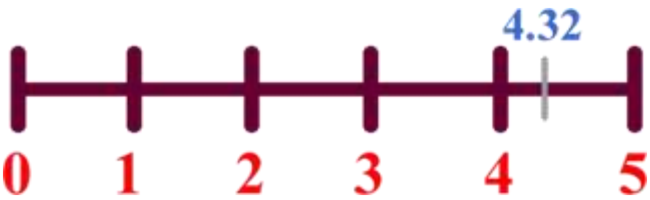

Fig 1

4.2 The identity $(\alpha+\beta)^2$

Most of the students in the 3rd class of Gymnasium usually make the following mistake: $(\alpha+\beta)^2 = \alpha^2+\beta^2$. This mistake can be somehow interpreted by Brousseau (1997, 2008) [3, 4], who pertains that an obstacle is not a lack of knowledge, but a knowledge, that produces appropriate answers within a frequently answered but limited context. This knowledge is not generalizable beyond this limited context in which it is valid.

This means that it fails to work efficiently in another context resulting in contradictions. Although students know that $(a*b)^2=\alpha^2*\beta^2$ is true, in fact when they are in the context of multiplication they generalize it and transfer it to addition, thus making a mistake. Indeed, students can be helped to overcome this cognitive obstacle with the help of the figures, that depict exactly how students were taught the same knowledge when they were in primary school. Specifically, the students learned about the shapes in the primary school, i.e they learned about the square and the rectangle. Now in Gymnasium, they can express the areas of squares and rectangles with sides $\alpha+\beta$, α and β, i.e they express the following areas: $(\alpha+\beta)^2$, α^2, and β^2. To achieve this successfully, primary school students must have sufficiently learned to handle these figures. Consequently, teachers, in primary school, are encouraged to ask students to play with the squares and rectangular pieces in order to construct new shapes and, in particular to construct a square. It is already well established that students of Gymnasium systematically misunderstand certain concepts in mathematics, due to the existence of cognitive obstacles, like the one mentioned before. Primary school teachers should promote the use of the appropriate advance organizers to facilitate the procedure of overcoming these obstacles.

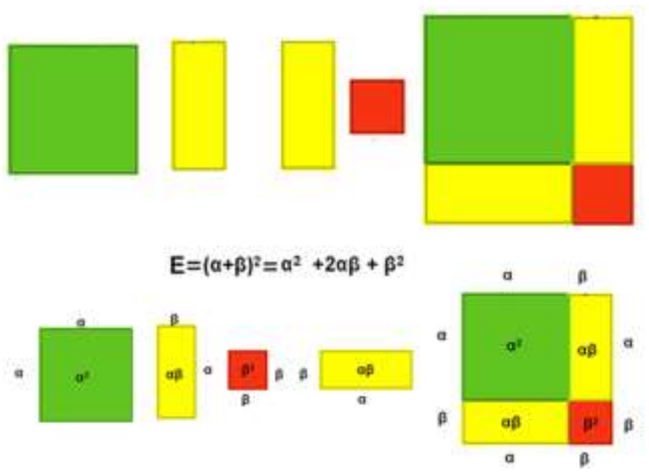

Fig 2

4.3 Converting units of measurement

The conversion of measurement units are quite difficult, not only for primary education students, but for secondary education students, too. For primary school students, things seem to be even more difficult. Here we propose a method to help novice primary school students to learn how to convert a meter to decimeters, centimeters and millimeters. We suggest that, the critical characteristic that will contribute to the establishment of the appropriate context for the novice students to understand is to motivate them to play with the subdivisions of the measure. Specifically, using bricks of one millimeter (magnets) of the same color e.g. red that will create the centimeter, decimeter and meter. The students, while playing, realize the relationship of the millimeter with the centimeter, with the decimeter and the meter. The students then, by substituting every ten millimeters, i.e. every 10 red bricks with a brick one centimeter long of another color e.g. blue brick, they understand that replacing all dozens with blue bricks results in 100 blue bricks and corresponding to the decimeters. It is obvious that we will need four colors of bricks. In this way, the student, by constructing the subdivisions of the measure himself, learns at the same time their relationship. We also note that each time the students make any length, they can recognize the bricks they use for it. So by using red for mm, blue for cm, green for dm and yellow for measure, then they can express in colors any length. For example, to express the length of 1.23 m, you will need 1230 red bricks! So, if asked what they prefer, they will prefer the 123 blue bricks instead of the 1230 red or one yellow and 23 blue bricks or 12 green and 3 blue. Depending on the size of the yard, only one or two toys will fit. This is basically a game of understanding a difficult concept and process, i.e. this conversion of units. The construction of one-dimensional, two-dimensional and three-dimensional objects that the student must have, in order to face the geometric objects of the space in the secondary education.

5. Conclusions

The process of teaching and learning is a complicated and multi-faceted dynamic procedure, where multiple factors of varied significance interact simultaneously, thus resulting in an unpredictable output. Changes in teachers' beliefs regarding a subject - mathematical, epistemological, or didactic - always represent an issue which is not easy to confront, because, in some cases, they come into conflict with sensitive personal and professional aspects (Sbaragli et al, 2011) [8]. This happens, because the beliefs of teachers, both cognitive and didactic, seem to define the classroom activities and also tend to influence their interpretation of their role; i.e what to teach, how and why. Moreover, these beliefs have considerable didactic significance (Schoenfeld, 1983) [10]. Also, beliefs can be an obstacle, but also a powerful force which allows the carrying out of changes in teaching (Tirosh, Graeber, 2003) [14]. Amongst the many possible causes, our research clearly shows the influence exerted by teachers beliefs, which, as has been seen, determine both the beliefs of the students, being in some cases responsible for their failure to develop significantly over time, and also for the teachers' fear of proposing sufficiently varied and rich situations, thus adding didactic obstacles, that are avoidable to the already existing (objective) epistemological obstacles. It seems that the notion of an obstacle is relative and that an obstacle may manifest itself to a younger student while remains unnoticed by an adult. The proposed didactic methods and measures consist not so much in avoiding obstacles as in conquering them. What we consider as the most important conclusion of this paper is that certain knowledge that represents obstacles for teachers, should be highlighted and adapted appropriately so that can be advance organizers for the knowledge of next level of education.

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