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Moments and couples

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Abstract

Moments are usually defined with respect to a fixed reference point; they deal with physical quantities as measured at some distance from that reference point. For example, the moment of force acting on an object, often called torque, is the product of the force and the distance from a reference point. In principle, any physical quantity can be multiplied by distance to produce a moment; commonly used quantities include forces, masses, and electric charge distributions.

When a force acts on a particle, the motion that can occur is that of translation only i.e., the particle moves in a straight line. But forces acting on a rigid body may produce in it the following types of motion:

1. Motion of Translation
2. Motion of Rotation
3. A combination of both

The tendency of a force to produce rotation about a fixed point is directly proportional to the product of the force P and the perpendicular distance p, of the line of action of the force from O.

This product P × p is called the moment of the force P about O.The resultant force exists of two unequal and unlike parallel forces, with different line of actions acting on a rigid body. But if two equal and unlike parallel forces with different line of actions act on a rigid body, then the resultant force of the two cannot be found by combining these forces. Equivalently, no single force can replace two equal and opposite forces with different line of action. These kind of equal and opposite forces are said to form a couple.

Keywords: moments, couples, motion of translation, motion of rotation

Introduction

Definition: The moment of a force about a point is the product of the force and the perpendicular distance of its line of action from the point.

Thus moment of a force P about O = P × p.

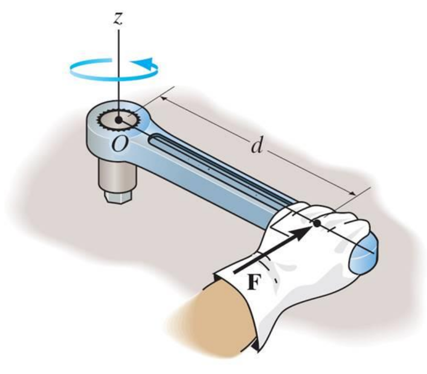


Fig 1

The tendency of the rotation of a body about a point can be in the clockwise or anti-clockwise direction.

Generally, the moment is regarded positive if the force P has a tendency to rotate the body about the fixed point O in the anti-clockwise direction.

The moment is said to be negative if the direction of rotation is clockwise. The moment of a force about a point is represented geometrically by twice the area of the triangle whose base is the line representing the force and whose vertex is the given point.

Varignon's theorem statement

The algebraic sum of the moments of two coplanar forces (not forming a couple) acting on a rigid body about any point in their plane is equalto the moment of their resultant about that point.

Generalization of varignon's theorem of moments statements

If any number of co-planar forces acting on a rigid body have a resultant, the algebraic sum of their moments about any point in their plane is equal to the moment of their resultant about that point.

Proof. Let P_1, P_2, P_3, P_4, ... P_n be n coplanar forces acting on a rigid body and O be the point about which moments are to be taken. Let R be their resultant.

Suppose no two above forces form a couple.

Let R_1 be the resultant of P_1 and P_2 ;

R_2 that of R_1 and P_3 i.e., of P_1, P_2, P_3;

R_3 that of R_2 and P_4 i.e., of P_1, P_2, P_3, P_4; and so on

Finally, let R_{n-1} be the resultant of R_{n-2} and P_n i.e., of P_1, P_2, P_3, ... P_n.

∴ By Varignon's theorem of moments,

Moment of R_1 about O = moment of P_1 about O + moment of P_2 about O Moment of R_2 about O = moment of R_1 about O + moment of P_3 about O

= moment of P_1 about O + moment of P_2 about O + moment of P_3 about O

and so on.

Finally, the moment of R_{n-1} about O

= algebraic sum of moments of R_{n-2} and P_n about O i.e., moment of R about O = algebraic sum of moments of forces P_1, P_2, P_3, ..., P_n about O Hence the result.

Conversely: If the algebraic sum of the moments of a system of coplanar forces (not equivalent to a couple) acting on a rigid body about a point in their plane is zero, then either the system of forces is in equilibrium or their resultant passes through that point. (Since $R \times p = 0$, then either $R = 0$ or $p = 0$)

A system of coplanar forces acting on a rigid body can be reduced to either a single force or a single couple

Two intersecting forces can be combined into a single resultant by parallelogram law of forces. Again, two parallel forces (not forming a couple) can be reduced to a single force by the law of parallel forces.

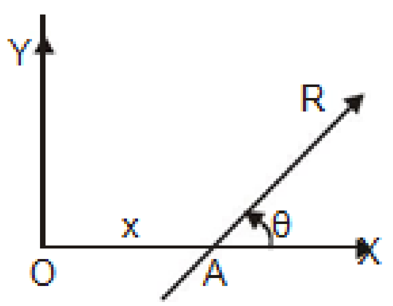
Let F_1, F_2, F_3 be any three forces of the system. Out of these atleast two are such that they do not form a couple, say, F_1 and F_2. Let R_1 be their resultant. Thus the three given forces F_1, F_2, F_3 have been reduced to two forces R_1 and F_3.

Again take a fourth force F_4 of the system. The three forces R_1, F_3, F_4 can be reduced to two forces as before, and so on. Repeating this process, in the end we are left with two forces which either form a couple or give a single resultant force.

Hence the system can be reduced to either a single force or a single couple.

Resultant of a system of coplanar forces acting on a rigid body

Let OX and OY be two perpendicular directions and P_1, P_2, P_3, ..., P_n be n coplanar forces acting on a rigid body making angles α_1, α_2, α_3, ..., α_n respectively with OX. Let R be their resultant making an angle θ with OX.



Resolving the forces along OX, we have

$R \cos\theta = P_1 \cos\alpha_1 + P_2 \cos\alpha_2 + ... + P_n \cos\alpha_n = X$ (say) ...(1)

Resolving the forces along OY, we get

$R \sin\theta = P_1 \sin\alpha_1 + P_2 \sin\alpha_2 + ... + P_n \sin\alpha_n = Y$ (say) ...(2)

Squaring (1) and (2) and adding, we get

$R^2 = X^2 + Y^2$ which gives the magnitude of the resultant

Again, dividing (2) by (1), we have $\tan\theta = \frac{Y}{X}$, which gives the direction of the resultant

To determine the line of action of the resultant, let us suppose that it meets OX in A at a distance x from O.Since A lies on the line of action of the resultant, the algebraic sum of moments of forces about A = 0, which gives us the value of x and hence the position of A.

Hence the resultant is a force of magnitude R and its line of action passes through A, at a distance x from O and makes an angle θ with OX.

Definition: Two equal and unlike parallel forces with different lines of action are said to form a couple.

The force applied to a key of a clock in order to wind it up or to a key of a lock to lock or unlock it are some examples of a couple.

The moment of couple is defined as the product of the magnitude of either of the forces forming the couple and the arm of the couple. i.e., Moment of couple (P, p) = P.p

= Force × Arm of the couple

Remark: The moment of a couple can never be zero.

The moment of a couple is said to be positive or negative according as it rotates the body in the anti-clockwise or clockwise direction.

The algebraic sum of the moments of two forces forming a couple about any point in their plane is constant.

Equilibrium of two couples

- Two coplanar couples of equal and opposite moments, balance each other.
- Two couples in the same plane of equal moments and acting in the same sense are equivalent (i.e., are equal to each other).
- Two couples of equal and opposite moments in parallel planes balance each other.
- The resultant of a number of coplanar couples is equivalent to a single couple whose moment is equal to the algebraic sum of the moments of the given couples.

A system of coplanar forces acting in one plane at different points of a rigid body can be reduced to a single force acting at any arbitrary point of the body together with the couple.

If three forces acting on a rigid body be represented in magnitude, direction and line of action by the sides of a triangle taken in order, they are equivalent to a couple whose moment is represented by twice the area of the triangle.

If three non-concurrent coplanar forces acting on a rigid body be equivalent to a couple, they must be proportional to the sides, taken in order, of the triangle formed by their lines of action.

Resultant of a force and a couple

A single force and a coplanar couple acting on a rigid body cannot produce equilibrium but are equivalent to a single force equal and parallel to the given force.

Resolution of a force into a force and a couple

Any force is equivalent to an equal and parallel force acting at an arbitrary point together with a couple of moment equal to the moment of the given force about that point.

References

1. Physics for Engineering by Hendricks, Subramony, and Van Blerk, 148. Web link
2. Dynamics, Theory and Applications by T.R. Kane and D.A. Levinson, 1985, pp. 90-99: Free download

3. Engineering Mechanics: Equilibrium, by C. Hartsuijker, J. W. Welleman, page 64 Web link
4. Augustus Jay Du Bois. The mechanics of engineering. Wiley. 1902; 1:186.
5. Ericksen JL. Timoshenko Acceptance Speech at iMechanica.org site for mechanicians. 1979.
6. Girvin HF. Applied Mechanics, §28 Couples, Scranton Pennsylvania: International Textbook Company, 1938; 33:4.